DIELECTRIC PROPERTIES OF SALT SOLUTIONS AT ULTRA-HIGH FREQUENCIES

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ABSTRACT The dielectric properties of (1) sodium chloride (2) magnesium chloride (3) potassium chloride and (4) copper sulphate solutions over a range of ultra-high frequencies (60—110 Mc/s) were studied by employing a Lecher-wire system terminating at the input end by a condenser filled with each of the solutions. The length of the Lecher-wire was adjusted for maximum voltage across the experimental condenser with the help of a valve-detector unit.

The dielectric constant of the solutions was found to increase, attaining a saturation value at a particular concentration (0.4—0.6%), except in the case of copper sulphate solution.

The ultra-high frequency absorption in each of the four salt solutions was studied by drawing the space-resonance curves with different concentrations of the salt solution. Absorption curves were then drawn by plotting the values of the width of resonance against concentration for several frequencies in the case of sodium chloride solution and for one frequency in the case of the other solutions. From these absorption curves, the frequencies of the maximum absorption for certain concentrations in the case of sodium chloride solution were found. From the observed values of the frequencies of maximum absorption the relaxation times were determined for these concentrations by applying Debye's formula. Relaxation time for a particular concentration of each of the other three solutions was also similarly obtained. The values of the product of the wavelength corresponding to the maximum absorption and the normality of the solution, expressed in gram-equivalents per litre, have been found to be constant which agree well with the values obtained by other workers. With the values of relaxation time obtained from the absorption data, Debye's dispersion, absorption and loss-tangent curves have been drawn for each of the four solutions. The values of relaxation time were also computed from Debye-Falkenhagen formula for comparison. In the case of copper sulphate, there was discrepancy.

The values of the high-frequency electrical conductivity of sodium chloride have also been calculated for different concentrations for three different frequencies within the experimental range.

INTRODUCTION

As a large part of wireless communication is conducted over the seas, the study of dielectric properties of seawater is of considerable importance for radio communications. From theoretical aspects also, the subject is worthy of systematic investigations. Experiments were therefore, undertaken by various workers for finding the dielectric properties of sea-water.

As early as 1907 the electrical conductivity of different samples of sea-water was measured by Hill (1907) in the audio-frequency range and later by Van der Pol (1918), in the range of low and medium frequencies. Drysdale (1920), studied the dielectric behaviour of sea-water up to a

frequency of 1 Mc/s and Smith-Rose (1933-34) carried out similar measurements up to 10 Mc/s

Sea-water mostly contains sodium chloride solutions in water and the salt-content varies from about 0.2 to about 3%. It also contains other salts in smaller quantities, viz., potassium chloride, magnesium chloride etc. (Dors, 1940). The dielectric behaviour of these salt solutions was also studied experimentally by different workers. Drake, Pierce and Dow (1930) measured the dielectric properties of potassium chloride solution at lower frequencies. Cooper (1946) measured the dielectric properties of sodium chloride solution over a frequency of 0.95–13 Mc/s and 690–4320 Mc/s. Some accurate measurements of the absorption coefficients of common salt in solution were made by Saxton and Lane (1947) for very high radio frequencies. The absorption and other associated properties of some salt solutions that compose average sea water were also studied recently by Chatterjee and Sreekantan (1948) in India over a frequency range of 300-500 Mc/s. An optical method was adopted in their work and the percentage of absorption of ultra-high frequency waves in these solutions as found for different concentrations over the experimental range of frequencies. Attenuation coefficients and hence absorption indices were calculated for different frequencies for certain fixed dilutions. Several other associated properties, e. g., relaxation time, dielectric constant, loss-tangent value, dipole-conductivity, etc. were also calculated from the absorption data for these solutions.

In the present work, the dielectric properties of sodium chloride, potassium chloride, magnesium chloride and copper sulphate solutions were investigated for a range of concentrations from 0 to 3% with ultra-high frequency fields ranging from about 90 Mc/s to 110 Mc/s. Regarding the effective dielectric constants of the solutions, no attempt was made to find their absolute values. Only a qualitative study of the variation of the dielectric constant of the different solutions with concentration was made for different frequencies within the experimental range. Within the same frequency range, measurements were made of H.F. absorption in water and in the different solutions. These absorption measurements were carried out for varying concentrations by keeping frequency constant and the results have been theoretically examined. The relaxation times have been computed from these absorption data, and with these computed values, the dielectric constant and the absorption coefficient have been obtained from the dispersion and absorption formulæ of Debye.

An expression for the H.F. electrical conductivity has been obtained in terms of absorption and its values have been obtained in the case of sodium chloride for different concentrations and for some fixed frequencies.

EXPERIMENTAL PROCEDURE

The Lecher-wire method was employed in studying the dielectric properties of the different salt solutions, viz. sodium chloride, magnesium chlo-

ride, potassium chloride, and copper sulphate for a range of ultra-high frequencies. The experimental arrangement consisted of a pair of Lecher wires terminating at the generator end by a parallel-plate condenser inside a glass receptacle for containing water or any solution as dielectric between

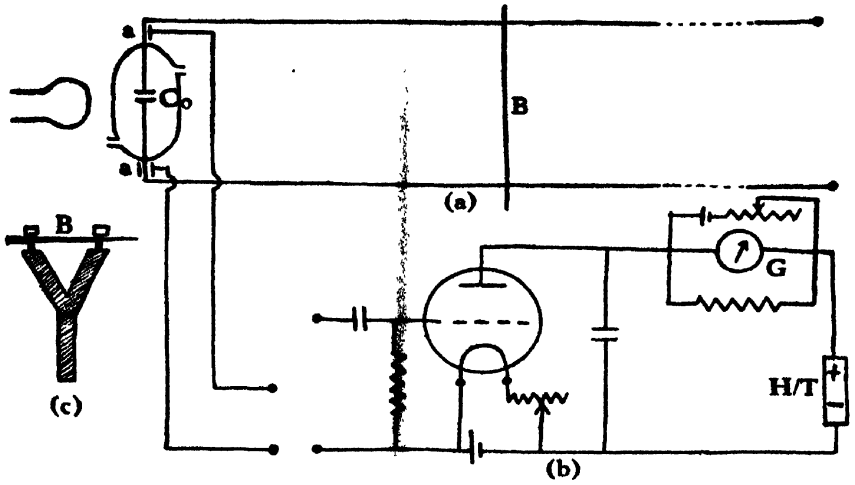

FIG. 1

(a) Lecher-wire arrangement
(b) Detector unit
(c) Short-circuting bridge.

the plates. When ultra-high frequency voltage was applied to the input end of the Lecher-wire system and standing waves were produced, the position of a potential node was obtained by sliding a short-circuiting bridge to-and-fro along the wires till the voltage across the terminal condenser as indicated by a suitable valve-detector unit was maximum*

DIELECTRIC CONSTANT VARIATIONS WITH CONCENTRATION

The length l of the Lecher wire (as measured from the generator end) was first adjusted for resonance with air in the experimental condenser. When water or any solution was introduced in the experimental condenser, the length l', when adjusted for resonance, was found to be shorter than the previous length l. The shortening indicated an increase of dielectric constant of the medium inside the condenser. The shift of resonance point was noted for different concentrations (0−3%) for each of the four salt solutions under examination. In the case of sodium chloride solution, five different frequencies from 62 Mc/s to 110 Mc/s were tried, while for other solutions, the

* It should be noted that the condition for maximum voltage across the terminal condenser was the same as that for current-resonance. The voltage-resonance adjustment was not only found convenient but also extremely useful, as it was possible to eliminate any conductivity effect across the experimental condenser. It is well known that the L-C value necessary for maximum voltage across the condenser should be independent of the conductivity or leakage across the same.

variation of the shift of resonance with concentration was studied for only one frequency within the experimental range.

H.F. ABSORPTION MEASUREMENTS

The study of the H.F. absorption in water and in the different solutions was done by drawing the space-resonance curves with pure water and with different amounts of salts dissolved in it. The "sharpness" or "flatness" of resonance obtained with water or any solution was then measured by noting the distance between two points on either side of the resonance point on the Lecher wire, giving currents $1/\sqrt{2}$ times the current at resonance.

Keeping the frequency of oscillation fixed, the width of resonance, as a measure of H.F. absorption, was measured from each of the space-resonance curves obtained with the solutions of different concentrations from 0 to 3%. With sodium chloride solution, fresh sets of observations were made for different frequencies within the range under investigation. Measurements were made with one frequency only for the other solutions.

The values of H/F electrical conductivity were determined from the absorption data in the case of sodium chloride solution for different concentrations $0-3\%$) for three fixed frequencies in the manner described in the next section.

THEORY OF THE METHOD OF MEASURING THE H.F. ELECTRICAL CONDUCTIVITY OF WATER AND THE SALT SOLUTIONS

The measurement of the H.F. electrical conductivity of the liquid between the plates inside the cell is possible by determining the attenuation constants of a Lecher-wire system (1) when there is between the plates of the terminal condenser and (2) when the liquid (water or any solution) is there as the dielectric. The attenuation constant α_0 in the first case is easily obtained from

$$\alpha_0 = \frac{R}{2Z_0} \qquad \ldots \quad (1)$$

where R is the H.F. resistance per unit length and Z_0 the surge-impedence of the Lecher system. The resistance R per unit length can be determined from

$$R = \sqrt{\frac{R_0\omega}{1-(d/a)^2}} \qquad \ldots \quad (2)$$

where R_0 denotes the direct-current resistance per unit length of the parallel wires, d the diameter of the wires and a the spacing. The units in (2) are in e.m.c.g.s. units. The surge-impedence is obtained from

$$Z_0 \text{ (ohms)} = 276 \log_{10}\frac{2a}{d} \qquad \ldots \quad (3)$$

The attenuation constant α, in the second case when water or any solution is inside the terminal condenser of the Lecher-wire system, can be determined in the following manner.

With the liquid inside the experimental condenser there will be a leakage of resistance ρ across the same. This leakage resistance is in parallel with the liquid condenser and can be replaced by a series resistance $r.\,l'$, where r = resistance per unit length distributed uniformly over the entire resonance length l'. The attenuation constant under this condition is given by

$$\alpha = \frac{R + r}{2Z_0} \qquad \ldots \quad (4)$$

Thus from (1) and (4)

$$r = 2Z_0(\alpha - \alpha_0)$$

or

$$r.l' = 2Z_0(\alpha - \alpha_0)l' \qquad \ldots \quad (5)$$

Now the leakage resistance is given by

$$\rho = \frac{1}{\omega^2 e^2 C_0^2 (rl')} \qquad \ldots \quad (6)$$

The capacity C_0 of the terminal condenser can be eliminated from the resonance condition, when there is just air between the plates of the terminal condenser of the Lecher system, *viz.*,

$$Z_0 . \tan \frac{2\pi l}{\lambda} = \frac{1}{\omega C_0}$$

or

$$C_0 = \frac{1}{\omega Z_0 \tan \frac{2\pi l}{\lambda}}$$

Substituting this value of C_0, we obtain from (5) and (6)

$$\rho = \frac{Z_0 \tan^2 \frac{2\pi l}{\lambda}}{2(\alpha - \alpha_0)\, e^2 l'} \qquad \ldots \quad (7)$$

Remembering that

$$\rho = \frac{1}{4\pi\sigma C_0}$$

and substituting the value of C_0, we get

$$\rho = \frac{\omega Z_0 \tan \frac{2\pi l}{\lambda}}{4\pi\sigma} \qquad \ldots \quad (8)$$

From (6) and (7)

$$\sigma = \frac{\omega(\alpha - \alpha_0)l'}{2\pi \tan \frac{2\pi l}{\lambda}} e^2 = \frac{c(\alpha - \alpha_0)l'.e^2}{\lambda \tan \frac{2\pi l}{\lambda}} \qquad \ldots \quad (9)$$

where c is the velocity of light and λ, the wavelength corresponding to ω.

Since the attenuation constant α_0, as seen from (1), is negligible, the electrical conductivity σ is given by

$$\left[\frac{\epsilon^2 l' C}{\lambda \tan \frac{2\pi l}{\lambda}}\right] \alpha \quad k_1 \alpha \qquad \ldots \quad (10)$$

where

$$k_1 = \epsilon^2 l' C / \lambda \tan \frac{2\pi l}{\lambda}$$

The attenuation constant α is obtained from the theoretical relation between α and the ' width ' of the resonance curve. In the case of ' half-width ', the attenuation coefficient is given by

$$\alpha = \frac{2}{m}\left(\frac{\pi e}{\lambda^2}\right) \qquad (11)$$

where e is the half-width, m is the order of the potential node on the Lecher wire and λ the wave-length of the oscillation induced into the Lecher-wire system. In the case of the $(1/\sqrt{2})$ width as in the present work, we have

$$\alpha = \frac{\sqrt{2}}{m}\left(\frac{\pi . e}{\lambda^2}\right) \qquad \ldots \quad (12)$$

where e is the $(1/\sqrt{2})$ width.

In view of (10) and (12), the H.F. conductivity will be given by the relation :

$$\sigma = k_1\left(\frac{\sqrt{2}}{m} \cdot \frac{\pi}{\lambda^2}\right)e$$

$$= \left[\frac{\epsilon^2 l' C}{\tan \frac{2\pi l}{\lambda}} \cdot \frac{\pi\sqrt{2}}{m\lambda^3}\right]e \qquad \ldots \quad (13)$$

EXPERIMENTAL ARRANGEMENTS

A Philco H.F. signal generator (Type 7070)) was used as source of ultra-high frequency oscillations. The output of the signal generator, when set for requisite ultra-high frequency, was applied across the parallel-plate condenser which served as the terminal condenser of a parallel-wire Lecher system at the generator end.

The pair of parallel Lecher-wires (S.W.G. No. 12 copper wire, diameter 0.259 cms.) were set up horizontally about 4 feet from the floor. The distance between the pair of parallel wires was 6.29 cms. As already mentioned above, the terminal condenser of the Lecher-wire system was a parallel-plate condenser enclosed in a glass receptacle with lead-in wires from the two plates. The glass receptacle could be filled with water or any other solution which also could be made to run out through an outlet tube fitted with a glass stop-cock. With ultra-high frequency oscillations on, the resonance length adjustments were made by sliding a short-circuiting bridge B across the Lecher-wires.

The short-circuiting bridge was made of thick copper wire and was mounted on a long ebonite handle. The under-side of the short circuiting bridge was filed and shaped in the form of a sharp edge and a pointer was attached to the bridge. A scale was kept horizontally fixed alongside the Lecher-wires and the pointer on the sliding bridge indicated its position on the horizontal scale. The resonance was indicated by means of a valve-detector with a balanced mirror galvanometer G in the anode circuit. The detector unit was coupled to the Lecher-wire system at the input end by means of two small condensers a, a consisting of short brass tubes insulated from the Lecher-wires by glass sleeves. The diagram of the entire experimental arrangement is shown in Fig. 1. The circuit diagram of the detector unit is shown separately in the same figure.

EXPERIMENTAL RESULTS SHOWING SHIFTS OF RESONANCE POINTS FOR DIFFERENT CONCENTRATIONS OF THE SOLUTIONS DUE TO DIELECTRIC CONSTANT VARIATIONS.

The shifts of resonance point on the Lecher-wires towards the generator end indicated smaller resonance lengths and hence increasing dielectric constant values. It was observed that the resonance length decreased gradually with the increased concentration, attaining a constant value after a certain concentration.

The shifts of resonance point for sodium chloride for increasing concentration from 0 to 3% (normality from 0 to 0.5129 N) for five different ultra-high frequencies are shown in Fig. 2. The shifts of resonance points

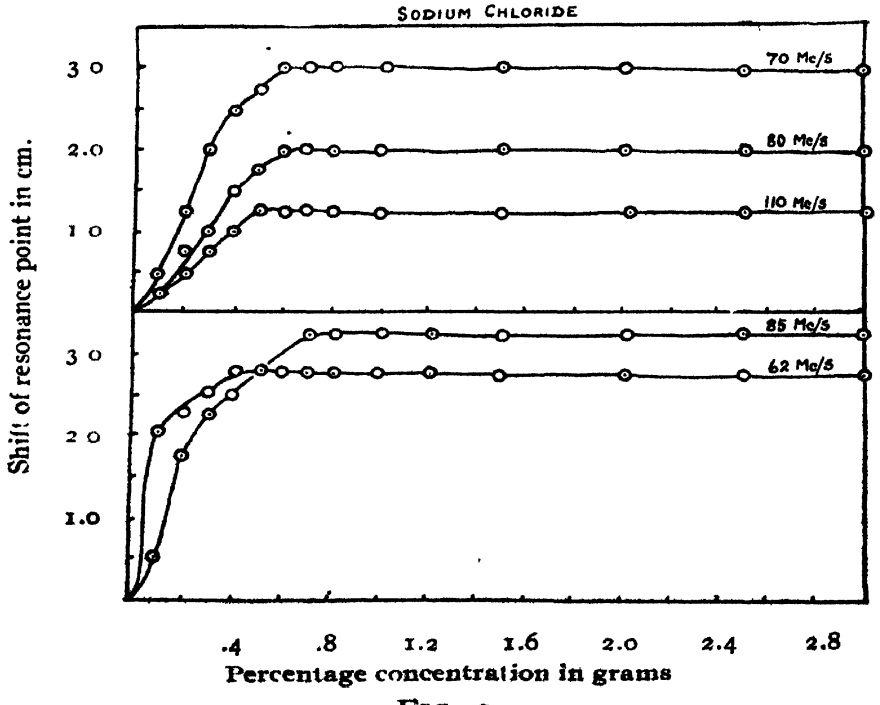

FIG. 2

for magnesium chloride, potassium chloride, and copper sulphate solutions with increasing concentration from o to 3% are illustrated in Fig. 3 for only one frequency.

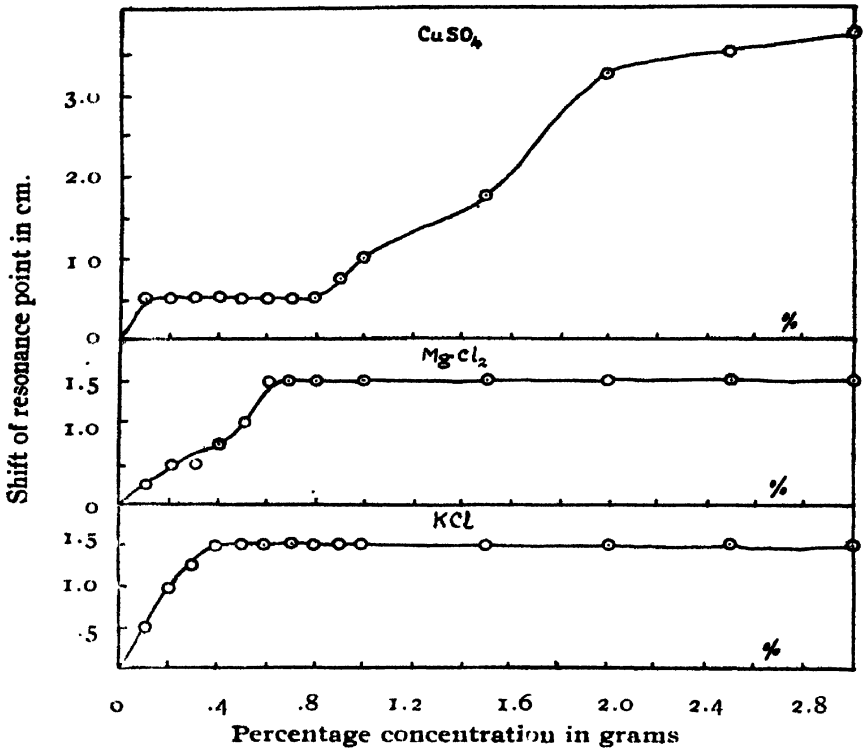

FIG. 3

It will be noticed that in all solutions, except copper sulphate solution, the shift increases gradually (*i.e.*, the resonance length l' decreases slowly) till it attains a steady value at a particular concentration. The maximum shift observed was indeed small, indicating only a small increase of dielectric constant of the solution. The dielectric constant attained a saturation value at 0.5 to 0.6% concentration. In the case of copper sulphate solution the dielectric constant was, however, found to remain constant from o to 0.8% concentration. For concentration values from 0.8 to 3%, it was found to increase considerably.

ABSORPTION MEASUREMENTS AND DETERMINATIONS OF H.F. ELECTRICAL CONDUCTIVITY OF THE SALT SOLUTIONS.

(a) *Absorption-concentration curves for fixed frequencies*

For each frequency, resonance curves showing deflections in the galvanometer of the valve-detector unit against the scale divisions representing the wire-lengths were experimentally obtained with water and with solutions of different concentration used as dielectric between the plates of the experi-

mental condenser. From the resonance curves for each concentration, the $(1/\sqrt{2})$ width was measured carefully. The values of the width of resonance when plotted against the corresponding values of concentration gave the absorption-concentration curve for a particular frequency. Such absorption curves were obtained for different frequencies

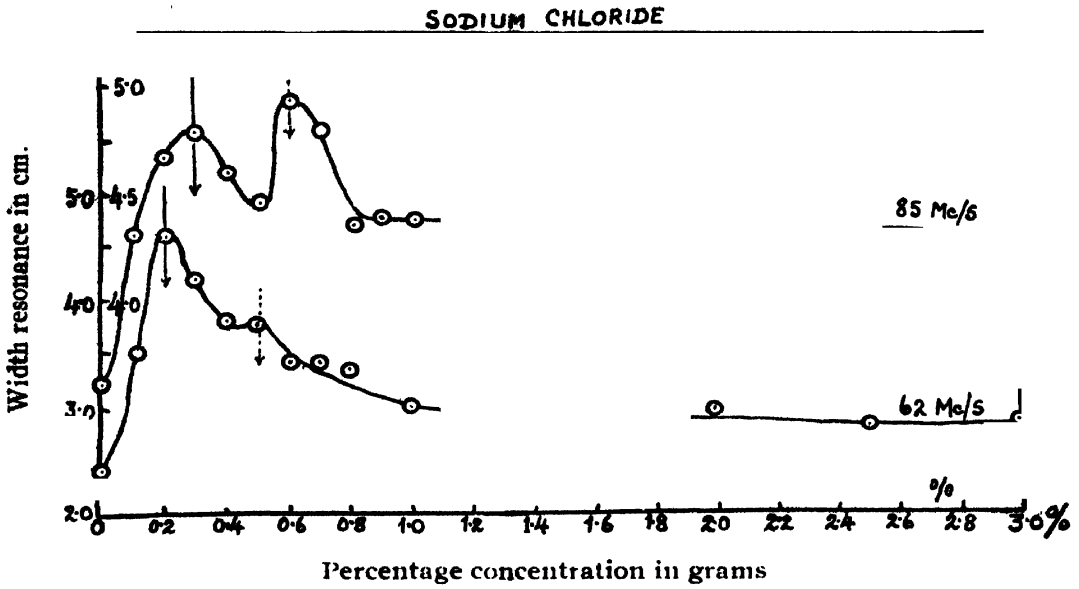

FIG. 4

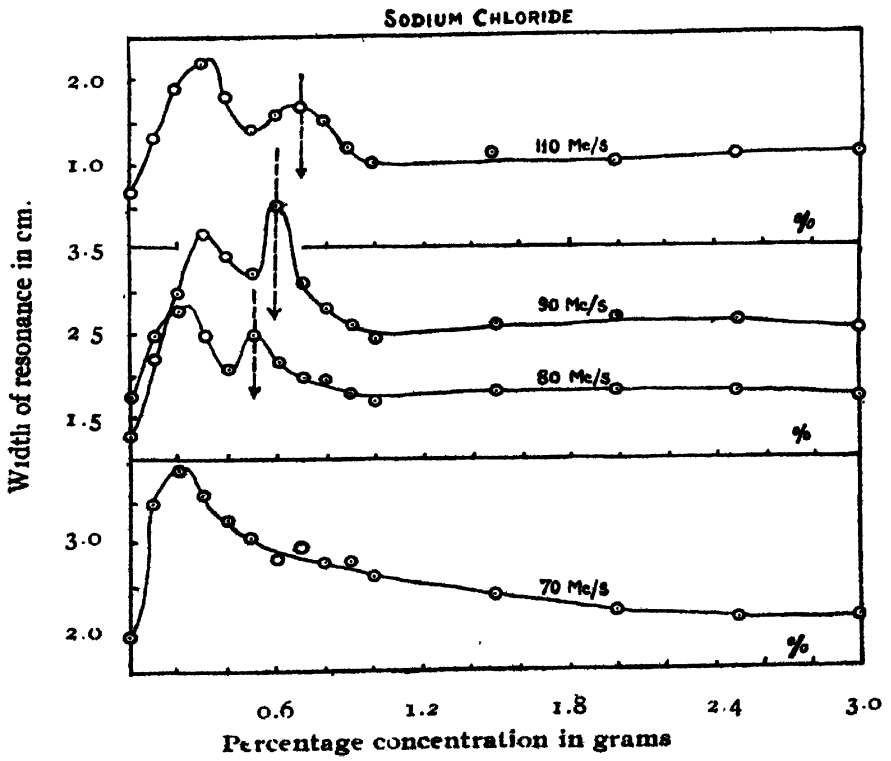

FIG. 5

in the case of sodium chloride solution, and for one frequency for other solutions. In almost all the curves there was distinct evidence of *two maxima* at particular values of the concentration, the positions of the maxima depending upon the frequency of the field. For sodium chloride solution two sets of absorption curves showing width of resonance against concentration are shown in Fig. 4 for 62Mc/s and 85 Mc/s. In Fig. 5 are shown four other sets of similar absorption curves for sodium chloride for 70 Mc/s, 80 Mc/s, 90 Mc/s and 110 Mc/s. The figures illustrate a definite shift of the maximum towards the higher concentration side for the higher frequencies. We have reasons to believe that the two maxima generally observed are associated with the fundamental and the first harmonic frequencies. It can be seen that the position of the first maximum (which is associated with the fundamental) lies between 0.2 and 0.3% conc. whereas, the position of the second maximum (which is associated with the first harmonic) lies within 0.5 and 0.7% conc. within the frequency range from 72 to 110 Mc/s. The shift of the second maximum with frequency is indicated in the absorption curves. The absorption-concentration curves for magnesium chloride, potassium chloride and copper sulphate solutions for one particular frequency are illustrated in Fig. 6.

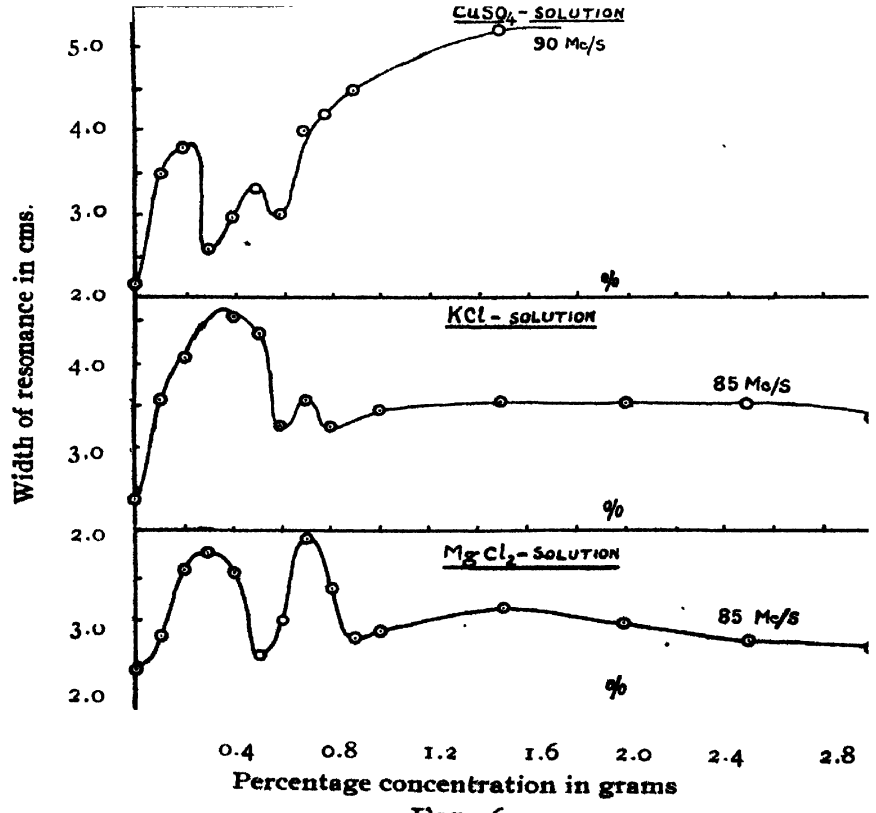

FIG. 6

In obtaining the values of the high-frequency electrical conductivity of he sodium chloride solution, which are given in Table I., calculations were

made according to (13). In computing the values, the dielectric constant of the aqueous solutions, which are all dilute, is taken as 80. The constant resonance-length l with air in the terminal condenser of the Lecher wire system and the resonance length l' with the solution inside the condenser after it attained a constant value were taken for the calculation.

TABLE I

(Sodium chloride solution)

% Concentration (grms.)	Normality (gms/litre)	H.F. elec. conductivity (e.s.u.)		
		70 Mc/s	80 Mc/s	90 Mc/s
0	0.0	0.07895	0.2303	0 4567
0.1	0.01709 N	0.10580	0.3036	0.7731
0.2	0.03419 N	0.15010	0.3350	1 0540
0.3	0.05129 N	0.13820	0.3036	1.3000
0.4	0.06834 N	0.12670	0 2617	1 1950
0.5	0.08547 N	0.11910	0.3036	1.1240
0.6	0.10260 N	0.10750	0.2722	1.4060
0.7	0.11960 N	0.11520	0.2513	1.0890
0.8	0.13680 N	0.10750	0 2513	0.9840
0.9	0.15380 N	0.10750	0.2303	0.9137
1.0	0.17090 N	0.09982	0.2199	0.8433
1.5	0.25640 N	0.09212	0.2303	0 9137
2.0	0.34130 N	0.84450	0 2303	0.9488
2.5	0.42740 N	0.08061	0.2303	0.9137
3.0	0.51290 N	0.08061	0.2303	0.8784

(*b*) *Frequency of Maximum Absorption and its Relation with Concentration*

From each absorption-concentration curve for a particular frequency, the concentration value for maximum absorption (*i.e.*, maximum width of resonance) was noted and the particular frequency was then regarded as the frequency for maximum absorption for that concentration. As already mentioned, there were two maxima in one absorption-concentration curve. The concentration values corresponding to the two maxima in the absorption curves were associated with the particular frequencies which were regarded as frequencies of maximum absorption for these concentrations. In Table II are given the different concentration values corresponding to frequencies

of maximum absorption for the two maxima. The values of the product of the wavelength in cms corresponding to the frequency of maximum absorption and the concentration expressed in gram-equivalent per litre are also entered in Table II.

TABLE II

(Sodium chloride solutions)

Set I

First maximum					Second maximum				
% conc. (gms)	γ gm. equiv. per litre	f Mc/s	$\lambda_{cm.}$	$(\gamma.\lambda_{cm.})$	% conc. (gms.)	γ gm. equiv. /litre	f Mc/s	$\lambda_{cm.}$	$(\gamma.\lambda_{cm.})$
0.20	0.0342 N	62	483 7	16.54	...	...	...	...	...
0.275	0.0470 N	85	352.9	16.59	0.6	0.1026	170	176 5	18.1
			mean	16.57					

Set II.

0.225	0.03846 N	70	428.6	16.48	...	...	..	...	...
0.250	0.04274 N	80	375.0	1..03	0.5	0.08547	160	177.5	16.03
0.300	0.05129 N	90	335.3	17.09	0.6	0.10260	180	167.7	17.09
c.325	0.05556 N	110	272.7	15.15	0.7	o 11960	220	136.4	15.15
				.....					.. ...
				mean value, 16.19					mean value, 16.48

It is to be observed that the product $(\gamma.\lambda_{cm})$ comes out to be a constant, being equal to 16.19 for the first mixinum. Accepting the first harmonic wavelength values for the second maximum, the product $(\gamma.\lambda_{cm})$ is found to have practically the same constant value. This equality of the values of $(\gamma.\lambda_{cm})$ for the two maxima can be regarded as a good justification for believing that the first hamonic is responsible for the second maximum.

The experimental value of $(\gamma.\lambda_{cm})$ for sodium chloride agrees well with the value 16.45 given by Forman and Crisp (1946) and also with that obtained by Chatterjee and Sreekantan (1948).

Similar experimental results with magnesium chloride, potassium chloride and copper sulphate solutions gave the frequencies of maximum absorption. The values of these frequencies and the corresponding concentrations together with the values of product of the wavelength corresponding to the frequency of maximum absorption and concentration in gram-equivalents per litre are given in the Table III.

TABLE III

		% Conc. (grms)	γ gm. equiv. per litre.	*f* Mc/s.	$\lambda_{cm.}$	$(\gamma . \lambda_{cm.})$
Potassium chloride.	1st max.	0.35	0.04693 N	85	352.9	16.56
	2nd max.	0.70	0.09386 N	170	176.5	16.56
						mean value 16.56
Magnesium chloride.	1st max	0.30	0.02950 N	85	352.9	10.41
	2nd max.	0.70	0.06885 N	170	176.5	12.15 mean-value : 11.28
Copper sulphate.	1st max.	0.275	0.02205 N	90	333.3	7.35
	2nd max.	0.50	0.04004 N	180	166.7	6.68
						mean value : 7.02

In Table IV the values of $(\gamma.\lambda_{cm.})$ obtained by different experimenters are given for comparison.

TABLE IV

Salt solution	Satyanarayana & Khastgir	Forman & Crisp	Chatterjee & Sreekantan
Sodium chloride	16.34	16.45	16.64
Potassium chloride	16 56	. .	15.96
Magnesium chloride	11.28	...	11.96
Copper sulphate	7.02	...	8.65

DISPERSION AND ABSORPTION FORMULÆ

The well-known Lorentz expression for the local field at a molecule is given by

$$E_{local} = E + 4\pi P \qquad \ldots \quad (14)$$

where P is the electric polarisation.

On the basis of this experssion, Debye (1929) derived the expressions for the real and imaginary dielectric constants :

$$\left.\begin{aligned} \epsilon' &= \epsilon_\infty + \frac{\epsilon_0 - \epsilon_\infty}{1+y^2} \\ \epsilon'' &= \frac{\epsilon_0 - \epsilon_\infty}{1-y^2} \cdot y \end{aligned}\right\} \quad \ldots \quad (15)$$

where $y = \left(\frac{\epsilon_0 + 2}{\epsilon_\infty + 2}\right)\omega\tau.$

ϵ_0 = static dielectric constant,

ϵ_∞ = dilectric constant for extremely high frequencies,

and τ = relaxation time.

The Lorentz expression was considered inadequate for representing the local field, and the hypothesis of 'hindered rotation' was introduced later by Debye (1935, 1937) and Fowler (1935). Reconsideration of the problem by Onsager (1936) led to a 'reaction field' which is due to the polarization of the dilectric in the electric field of the molecule itself. Considering the reaction field with certain simplifying assumptions and introducing the effect of an alternating field, Onsager derived

$$\left.\begin{aligned} \epsilon' &= \epsilon_\infty + \frac{\epsilon_0 - \epsilon_\infty}{1+x^2} \\ \epsilon'' &= \frac{\epsilon_0 - \epsilon_\infty}{1+x^2} \cdot x. \end{aligned}\right\} \quad \ldots \quad (16)$$

where $x = \omega\tau_0$ and τ_0 is another relaxation time.

Onsager's expressions were similar to Debye's except for the interchange of parameters x and y. This interchange of paremeters gives a larger value for the relaxation time according to Onsager than that on Debye's theory.

Van Vleck (1937) suggested an assemblage of discrete particles rather than a continuous medium. On this view Cole (1937) also obtained expressions similar to Debye's formulæ. The relaxation time was, however, different and there was an empirical factor.

EXPERIMENTAL DETERMINATION OF RELAXATION TIME FROM THE ABSORPTION MEASUREMENTS

Let us take Debye's expression for ϵ'' :

$$\epsilon'' = \frac{\epsilon_0 - \epsilon_\infty}{1+y^2} \cdot y,$$

where $y = \omega\tau \cdot \left(\frac{\epsilon_0 + 2}{\epsilon_\infty + 2}\right)$

When concentration is varied, both relaxation time and dielectric constant change, so that

$$\frac{\partial \epsilon''}{\partial y} = (\epsilon_0 - \epsilon_\infty) \frac{\partial}{\partial y}\left(\frac{y}{1+y^2}\right) + \frac{y}{1+y^2} \cdot \frac{\partial}{\partial y}(\epsilon_0 - \epsilon_\infty)$$

If now the dielectric constant is assumed to remain constant the second term becomes zero and ϵ'' becomes maximum, when $y=1$. The constancy of the dielectric constant has been assumed for determining the frequency of maximum absorption. Thus according to Debye

$$\omega_{max} = 2\pi f_{max} = \left(\frac{\epsilon_\infty + 2}{\epsilon_0 + 2}\right)\frac{1}{\tau}. \quad (y=1).$$

or

$$\tau = \frac{1}{2\pi f_{max}}\left(\frac{\epsilon_\infty + 2}{\epsilon_0 + 2}\right) \quad \ldots \quad (17)$$

According to Onsager

$$\tau_0 = \frac{1}{2\pi f_{max}}$$

In the range of ultra-high frequencies the difference between the two relaxation times is extremely small.

The calculation of τ from (17) has been primarily based on the observed value of the frequency of maximum absorption. ϵ_∞ may be assumed to be equal to the dielectric constant of water at ultra-high frequencies which is 80 and ϵ_0 at the audio frequencies can be obtained from Falkenhagen's relation :

$$\epsilon_0 - \epsilon_s = \frac{1.97 \times 10^6 \mid Z_1 Z_2 \mid \{\mid Z_1 \mid + \mid Z_2 \mid\}^{\frac{1}{2}} \cdot (q.\gamma)^{\frac{1}{2}}}{2\epsilon_s^{\frac{1}{2}} T^{\frac{3}{2}} (1 + 1/\sqrt{q})^2} \quad (19)$$

where Z_1 = valency of Na-ion = 1
Z_2 = valency of Cl-ion = 1
$T = 300°K$
ϵ_s = dielectric constant of water at u.h.f. = 80 e.s,u.
$q = 0.5$ for NaCl
γ = concentration in gram-equivalents per litre.

Thus $\epsilon_0 - \epsilon_s = 3.536\sqrt{\gamma}$ for NaCl-solution.

Similarly the following can be worked out.

$\epsilon_0 - \epsilon_s = 9.281\sqrt{\gamma}$ for $MgCl_2$ at 23°C.
$\epsilon_0 - \epsilon_s = 3.51\sqrt{\gamma}$ for KCl at 23°C.
$\epsilon_0 - \epsilon_s = 28.95\sqrt{\gamma}$ for $CuSO_4$ at 23°C

The relaxation time according to Debye-Falkenhagen theory (Falkenhagen, 1935) for a salt solution is given by :—

$$\tau = \frac{8.85 \times 10^{-11} \times D_v}{\Delta_\infty . \gamma} \quad (20)$$

where D_0 = dielectric constant of water
γ = concentration expressed in grain equivalents per litre,
Δ_∞ = equivalent conductivity of solution at infinite dilution,
= 108.99 for sodium chloride
= 110.88 for magnesium chloride
= 133.10 for potassium chloride
= 113.85 for copper sulphate

The values of ϵ_0, as calculated from (19), and of τ, as calculated from (17) are given in Table V in the case of NaCl for three different concentrations. The values of τ, as computed from Debye-Falkenhagen formula, are also entered in the same table for comparison.

TABLE V

Sodium chloride solution. $\epsilon_s = 80$

% conc. (gms).	γ normality (gms/litre)	ϵ_0	f_{max} Mc/s.	Relaxation time τ	
				Obs. in secs. $\times 10^{10}$ (Debye)	Cal. in sec. $\times 10^{10}$ (Debye-Falkenhagen)
0.7	0.11960 N	81.223	110	14.25	11.69
0.6	0.10260 N	81.132	90	17.44	12.67
0.5	0.58547 N	81.034	80	19 63	15.20

The values of ϵ_0 and τ for KCl, $MgCl_2$, and $CuSO_4$ for one concentration are given in Table VI.

TABLE VI

Salt	% conc. (gms.)	γ normality (gms/litre)	ϵ_0	f_{max} Mc/s.	Relaxation time τ	
					Obs in secs. $\times 10^{10}$ (Debye)	Cal. in secs $\times 10^{10}$ (Debye-Falkenhagen)
KCl	0.35	0.04693 N	80.761	85	19.01	11.60
$MgCl_2$	0 70	0 06885 N	82 435	85	18.18	21.79
$CuSO_4$	0.50	0.04004 N	85.794	90	16.84	28.19

It is to be observed that Debye-Falkenhagen expression for τ gives somewhat smaller values than our observed values for NaCl and KCl. In the case of $MgCl_2$, the observed value is slightly higher. The discrepancy between the observed value and the value computed from Debye Falkenhagen expression is indeed large for $CuSO_4$ solution.

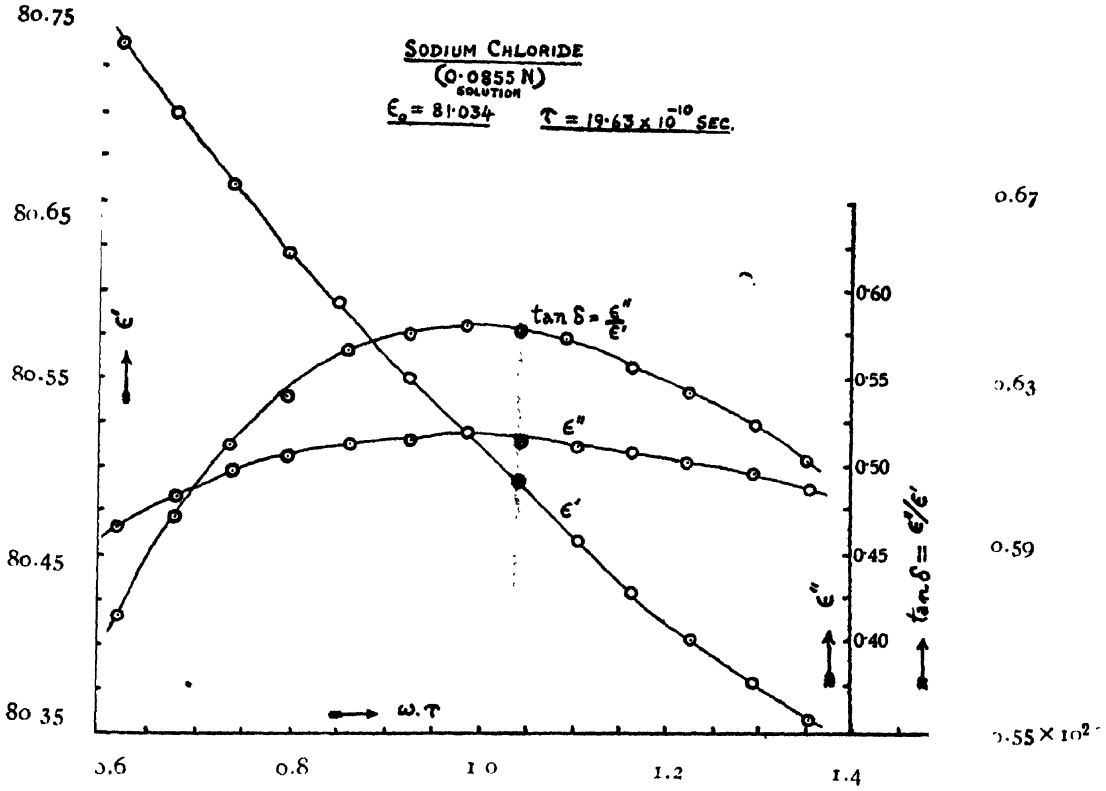
SODIUM CHLORIDE
(0·0855 N) SOLUTION
$\epsilon_0 = 81{\cdot}034$
$\tau = 19{\cdot}63 \times 10^{-10}$ SEC.
$\tan\delta = \frac{\epsilon''}{\epsilon'}$
ϵ'
ϵ''
$\omega.\tau$
80.75
80.65
80.55
80.45
80 35
0·60
0·55
0·50
0·45
0·40
0.67
0.63
0.59
0.55×10^{2}
0.6
0.8
1 0
1.2
1.4

FIG. 7

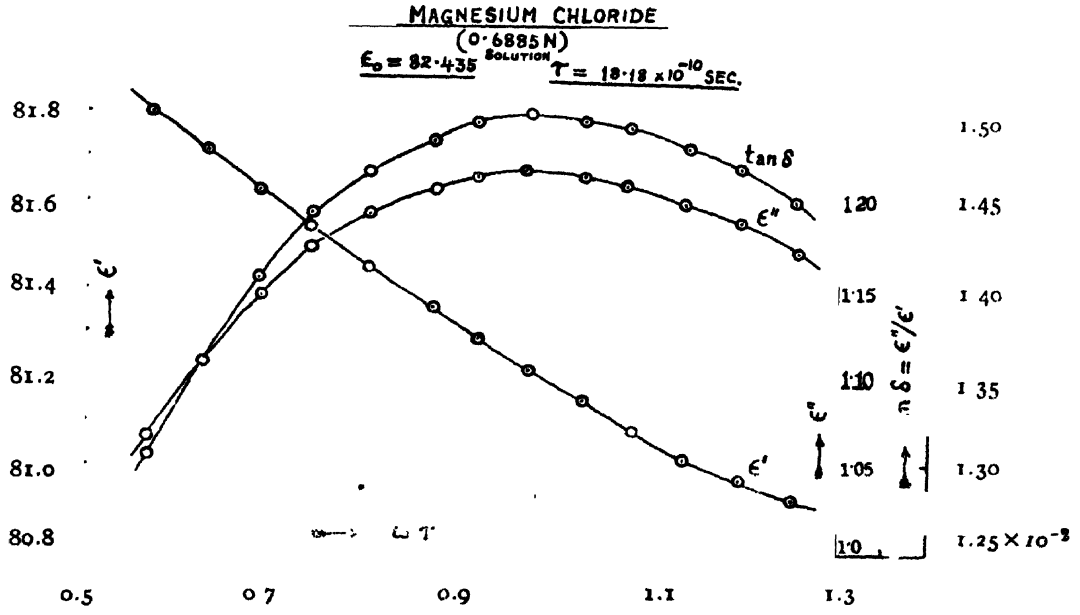
MAGNESIUM CHLORIDE
(0·6885 N) SOLUTION
$\epsilon_0 = 82{\cdot}435$
$\tau = 18{\cdot}18 \times 10^{-10}$ SEC.
$\tan\delta$
ϵ''
ϵ'
$\omega\tau$
81.8
81.6
81.4
81.2
81.0
80.8
1·20
1·15
1·10
1·05
1·0
1.50
1.45
1 40
1 35
1.30
1.25×10^{-2}
0.5
0 7
0.9
1.1
1.3

FIG.8

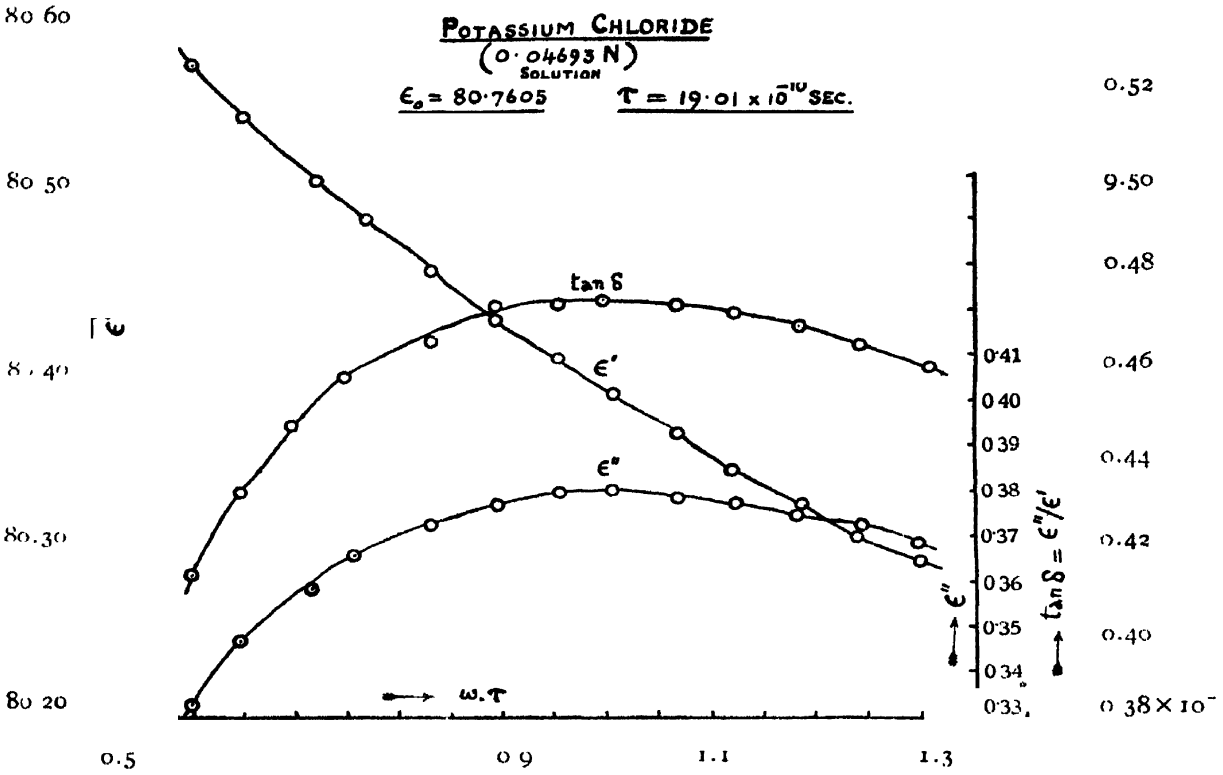
POTASSIUM CHLORIDE
(0·04693 N)
SOLUTION
ϵ_0 = 80·7605
τ = 19·01 × 10⁻¹⁰ SEC.
tan δ
ε'
ε''
ω.τ
ε'
ε''
tan δ = ε''/ε'
80 60
80 50
80 40
80 30
80 20
0·52
9·50
0·48
0·46
0·44
0·42
0·40
0·41
0·40
0·39
0·38
0·37
0·36
0·35
0·34
0·33
0·38 × 10⁻
0·5
0·9
1·1
1·3

FIG. 9

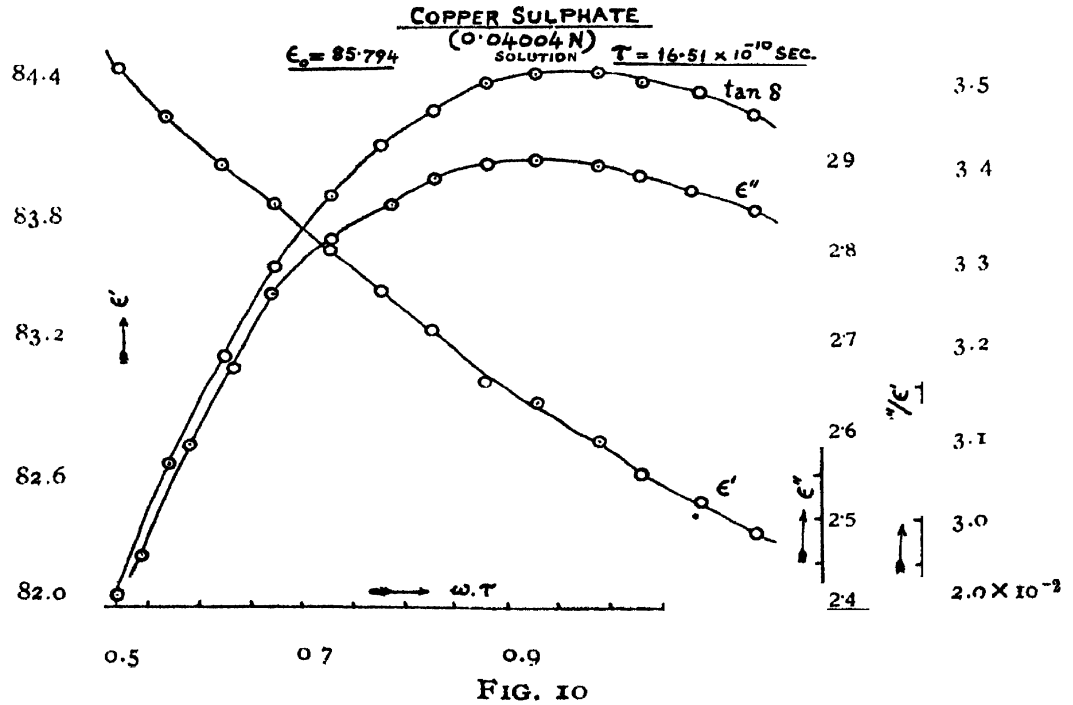
COPPER SULPHATE
(0·04004 N)
SOLUTION
ϵ_0 = 85·794
τ = 16·51 × 10⁻¹⁰ SEC.
tan δ
ε''
ε'
ω.τ
ε'
ε''
ε''/ε'
84·4
83·8
83·2
82·6
82·0
3·5
3·4
3·3
3·2
3·1
3·0
2·0 × 10⁻²
2·9
2·8
2·7
2·6
2·5
2·4
0·5
0·7
0·9

FIG. 10

CALCULATED VALUES OF ϵ', ϵ'' AND THE LOSS-TANGENT

Accepting Debye's formulæ for ϵ' and ϵ'', we have substituted our observed values of the relaxation time and have calculated ϵ' and ϵ'' for different ultra-high frequencies within the experimental range for one concentration value for each of the four salt solutions. The loss-tangent $\tan \delta = \frac{\epsilon'}{\epsilon''}$ has also been calculated. The variations of ϵ', ϵ'' and $\tan \delta$ with frequency are illustrated in figures 7 – 10. It will be seen that the maximum for either ϵ'' or $\tan \delta$ appears at the frequency of maximum absorption. It should be remembered that the values of ϵ'' and $\tan \delta$ were calculated by taking the relaxation time value corresponding to the frequency of maximum absorption.

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